

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 1

#### Question:

Whenever a numerical value of  $g$  is required take  $g = 9.8 \text{ m s}^{-2}$ .

Calculate the work done by a horizontal force of magnitude  $0.6 \text{ N}$  which pulls a particle a distance of  $4.2 \text{ m}$  across a horizontal floor.

#### Solution:

$$\begin{aligned}\text{Work done} &= F \times s \\ &= 0.6 \times 4.2 \\ &= 2.52\end{aligned}$$

The work done is  $2.52 \text{ J}$ .

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 2

#### Question:

A box is pulled 12 m across a smooth horizontal floor by a constant horizontal force. The work done by the force is 102 J. Calculate the magnitude of the force.

#### Solution:

$$\text{Work done} = F \times s$$

$$102 = F \times 12$$

$$F = \frac{102}{12} = 8.5$$

The magnitude of the force is 8.5 N.

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 3

#### Question:

Calculate the work done against gravity when a particle of mass 0.35 kg is raised a vertical distance of 7 m.

#### Solution:

$$\begin{aligned}\text{Work done against gravity} &= mgh \\ &= 0.35 \times 9.8 \times 7 \\ &= 24.01\end{aligned}$$

The work done against gravity is 24.0 J.

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 4

#### Question:

A crate of mass 15 kg is raised through a vertical distance of 4 m. Calculate the work done against gravity.

#### Solution:

$$\begin{aligned}\text{Work done against gravity} &= mgh \\ &= 15 \times 9.8 \times 4 \\ &= 588\end{aligned}$$

The work done against gravity is 588 J.

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 5

#### Question:

A box is pushed 15 m across a horizontal surface. The box moves at a constant speed and the resistances to motion total 22 N. Calculate the work done by the force pushing the box.

#### Solution:



$$F = 22$$

$$\begin{aligned}\text{Work done} &= F \times s \\ &= 22 \times 15 \\ &= 330\end{aligned}$$

The work done by the force pushing the box is 330 N.

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 6

**Question:**

A ball of mass 0.5 kg falls vertically 15 m from rest. Calculate the work done by gravity.

**Solution:**

$$\begin{aligned}\text{Work done by gravity} &= mgh \\ &= 0.5 \times 9.8 \times 15 \\ &= 73.5\end{aligned}$$

The work done by gravity is 73.5 J.

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 7

#### Question:

A cable is attached to a crate of mass 80 kg. The crate is raised vertically at a constant speed from the ground to the top of a building. The work done in raising the crate is 30 kJ. Calculate the height of the building.

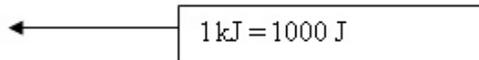
#### Solution:

$$\text{Work done} = mgh$$

$$30 \times 1000 = 80 \times 9.8 h$$

$$h = \frac{30\,000}{80 \times 9.8}$$

$$h = 38.26$$


$$1 \text{ kJ} = 1000 \text{ J}$$

The building is 38.3 m high.

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 8

#### Question:

A sledge is pulled 14 m across a horizontal sheet of ice by a rope inclined at  $25^\circ$  to the horizontal. The tension in the rope is 18 N and the ice can be assumed to be a smooth surface. Calculate the work done.

#### Solution:



$$\begin{aligned}\text{Work done} &= 18 \cos 25^\circ \times 14 \\ &= 228.3\end{aligned}$$

← Work done = *horizontal* component of tension  $\times$  *horizontal* distance moved

The work done is 228 J.

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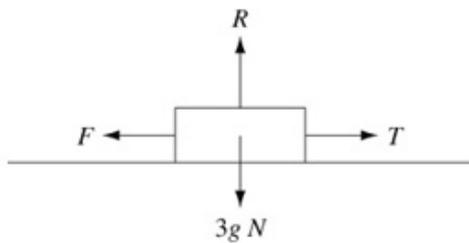
## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 9

#### Question:

A parcel of mass 3 kg is pulled at a distance of 4 m across a rough horizontal floor. The parcel moves at a constant speed. The work done against friction is 30 J. Calculate the coefficient of friction between the parcel and the surface.

#### Solution:



$$\text{Work done} = F \times s$$

$$30 = T \times 4$$

$$T = 7.5$$

$$F = ma$$

$$7.5 - F = 0$$

$$F = 7.5$$

$$R(\uparrow) \quad R = mg$$

$$R = 3 \times 9.8$$

$$\text{Friction is limiting} \quad F = \mu R$$

$$7.5 = \mu \times 3 \times 9.8$$

$$\mu = \frac{7.5}{3 \times 9.8} = 0.2551$$

The parcel moves at a constant speed so the acceleration is  $0 \text{ m s}^{-2}$ .

The coefficient of friction is 0.255.

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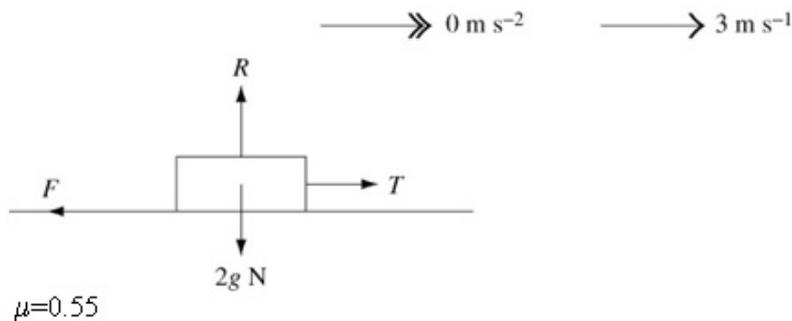
## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 10

#### Question:

A block of wood of mass 2 kg is pushed across a rough horizontal floor. The block moves at  $3 \text{ m s}^{-1}$  and the coefficient of friction between the block and the floor is 0.55. Calculate the work done in 2 seconds.

#### Solution:



$$R(\uparrow) \quad R = 2g$$

Friction is limiting

$$F = \mu R$$

$$F = 0.55 \times 2g$$

$$R(\rightarrow) \quad T - F = 0$$

$$T = 0.55 \times 2g$$

$$\text{Work done} = F \times s$$

$$= 0.55 \times 2g \times (3 \times 2)$$

$$= 0.55 \times 2 \times 9.8 \times 6$$

$$= 64.68$$

Distance moved = speed  $\times$  time

The work done is 64.7 J.

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 11

#### Question:

A girl of mass 52 kg climbs a vertical cliff which is 46 m high. Calculate the work she does against gravity.

#### Solution:

$$\begin{aligned}\text{Work done against gravity} &= mgh \\ &= 52 \times 9.8 \times 46 \\ &= 23\,441\end{aligned}$$

The work done against gravity is 23 400 J.

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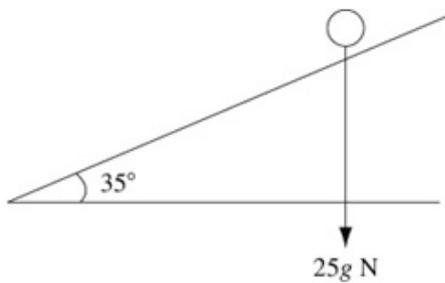
## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 12

#### Question:

A child of mass 25 kg slides 2 m down a smooth slope inclined at  $35^\circ$  to the horizontal. Calculate the work done by gravity.

#### Solution:



$$\begin{aligned}\text{Work done by gravity} &= mgh \\ &= 25 \times 9.8 \times (2 \sin 35^\circ) \\ &= 281.0\end{aligned}$$

Vertical distance  
moved =  $2 \sin 35^\circ$

The work done by gravity is 281 J.

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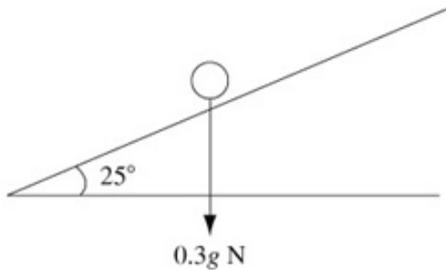
## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 13

#### Question:

A particle of mass 0.3 kg is pulled 2 m up a line of greatest slope of a plane which is inclined at  $25^\circ$  to the horizontal. Assuming that the particle moves along a line of greatest slope of the plane, calculate the work done against gravity.

#### Solution:



$$\begin{aligned}\text{Work done against gravity} &= mgh \\ &= 0.3 \times 9.8 \times (2 \sin 25^\circ) \\ &= 2.484\end{aligned}$$

The work done against gravity is 2.48 J.

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 14

#### Question:

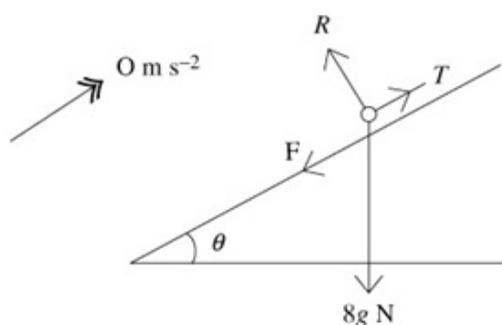
A rough plane surface is inclined at an angle  $\arcsin \frac{5}{13}$  to the horizontal. A packet of mass 8 kg is pulled at a constant speed up a line of greatest slope of the plane. The coefficient of friction between the packet and the plane is 0.3.

- a Calculate the magnitude of the frictional force acting on the packet.

The packet moves a distance of 15 m up the plane. Calculate

- b the work done against friction,  
c the work done against gravity.

#### Solution:

**a**

$$R(\sphericalangle) \quad R = 8g \cos \theta \\ = 8g \times \frac{12}{13}$$

Friction is limiting:

$$F = \mu R \\ F = 0.3 \times 8 \times 9.8 \times \frac{12}{13} \\ = 21.71$$

The frictional force has magnitude 21.7 N.

**b**

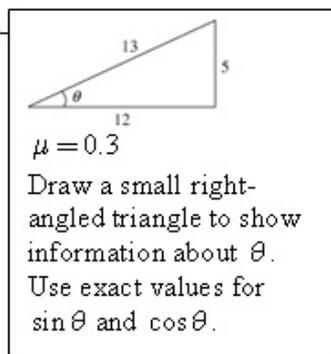
$$\text{Work done against friction} = F \times s \\ = 21.71 \times 15 \\ = 325.6$$

The work done against friction is 326 J.

**c**

$$\text{Work done against gravity} = mgh \\ = 8 \times 9.8 \times (15 \sin \theta) \\ = 8 \times 9.8 \times \left(15 \times \frac{5}{13}\right) \\ = 452.3$$

The work done against gravity is 452 J.



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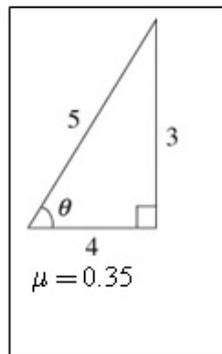
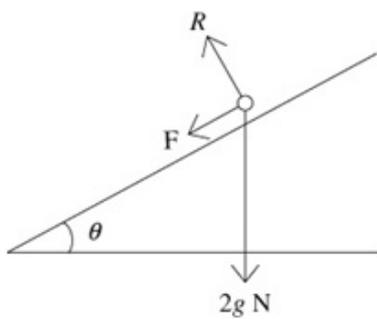
## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 15

#### Question:

A particle  $P$  of mass 2 kg is projected up a line of greatest slope of a rough plane which is inclined at an angle  $\arcsin \frac{3}{5}$  to the horizontal. The coefficient of friction between  $P$  and the plane is 0.35. The particle travels 3 m up the plane. Calculate the work done by friction.

#### Solution:



$$R(\sphericalangle) \quad R = 2g \cos \theta$$

$$R = 2 \times 9.8 \times \frac{4}{5}$$

Friction is limiting

$$F = \mu R$$

$$F = 0.35 \times 2 \times 9.8 \times \frac{4}{5}$$

$$\text{Work done} = F \times s$$

$$= 0.35 \times 2 \times 9.8 \times \frac{4}{5} \times 3$$

$$= 16.46$$

The work done by friction is 16.5 J.

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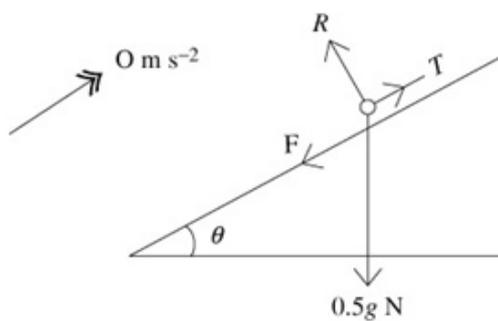
## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 16

#### Question:

A rough surface is inclined at an angle  $\arcsin \frac{7}{25}$  to the horizontal. A particle of mass  $0.5 \text{ kg}$  is pulled  $3 \text{ m}$  at a constant speed up the surface by a force acting along a line of greatest slope. The only resistances to the motion are those due to friction and gravity. The work done by the force is  $12 \text{ J}$ . Calculate the coefficient of friction between the particle and the surface.

#### Solution:



$$R(\searrow) R = 0.5g \cos \theta$$

$$= 0.5g \times \frac{24}{25}$$

$$R(\nearrow)$$

$$F + 0.5g \sin \theta = T$$

$$\text{Friction is limiting: } F = \mu R$$

$$F = \mu \times 0.5g \times \frac{24}{25}$$

$$T = \mu \times 0.5g \times \frac{24}{25} + 0.5g \times \frac{7}{25}$$

$$\text{Work done by force} = F \times s$$

$$12 = T \times 3$$

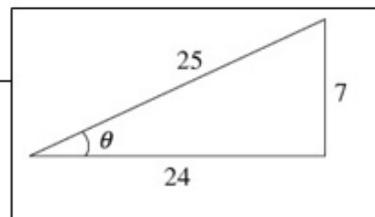
$$T = 4$$

$$\therefore 4 = \mu \times 0.5g \times \frac{24}{25} + 0.5g \times \frac{7}{25}$$

$$\mu = \frac{4 - 0.5 \times 9.8 \times \frac{7}{25}}{0.5 \times 9.8 \times \frac{24}{25}}$$

$$\mu = 0.5586$$

The coefficient of friction is 0.559.



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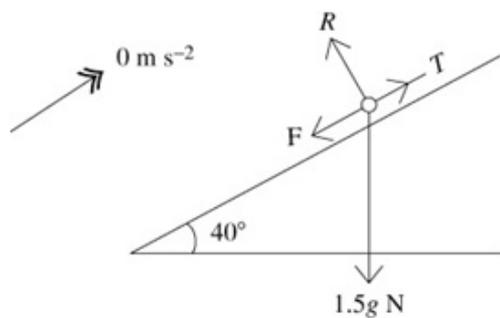
## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 17

#### Question:

A rough surface is inclined at  $40^\circ$  to the horizontal. A particle of mass  $1.5 \text{ kg}$  is pulled at a constant speed up the surface by a force  $T$  acting along a line of greatest slope. The coefficient of friction between the particle and the surface is  $0.4$ . Calculate the work done by  $T$  when the particle travels  $8 \text{ m}$ . You may assume that the only resistances to motion are due to gravity and friction.

#### Solution:



$$\mu = 0.4$$

$$R(\perp) R = 1.5g \cos 40^\circ$$

Friction is limiting:  $F = \mu R$

$$F = 0.4 \times 1.5g \cos 40^\circ$$

$$R(\parallel)$$

$$T = F + 1.5g \sin 40^\circ$$

$$T = 0.4 \times 1.5g \cos 40^\circ + 1.5g \sin 40^\circ$$

Work done by  $T = T \times s$

$$= (0.4 \times 1.5g \cos 40^\circ + 1.5g \sin 40^\circ) \times 8$$

$$= 111.6$$

The work done by  $T$  is  $112 \text{ J}$ .

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## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 1

#### Question:

Whenever a numerical value of  $g$  is required take  $g = 9.8 \text{ m s}^{-2}$ .

Calculate the kinetic energy of:

- a a particle of mass  $0.3 \text{ kg}$  moving at  $15 \text{ m s}^{-1}$
- b a particle of mass  $3 \text{ kg}$  moving at  $2 \text{ m s}^{-1}$
- c a box of mass  $5 \text{ kg}$  moving at  $7.5 \text{ m s}^{-1}$
- d an arrow of mass  $0.5 \text{ kg}$  moving at  $200 \text{ m s}^{-1}$
- e a boy of mass  $25 \text{ kg}$  running at  $4 \text{ m s}^{-1}$
- f a ball of mass  $0.4 \text{ kg}$  moving at  $15 \text{ m s}^{-1}$
- g a car of mass  $800 \text{ kg}$  moving at  $20 \text{ m s}^{-1}$

#### Solution:

$$\mathbf{a} \text{ K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.3 \times 15^2 = 33.75 = 33.8 \text{ J}$$

$$\mathbf{b} \text{ K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 2^2 = 6 \text{ J}$$

$$\mathbf{c} \text{ K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 7.5^2 = 140.625 = 141 \text{ J}$$

$$\mathbf{d} \text{ K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 200^2 = 10\,000 \text{ J}$$

$$\mathbf{e} \text{ K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 4^2 = 200 \text{ J}$$

$$\mathbf{f} \text{ K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.4 \times 15^2 = 45 \text{ J}$$

$$\mathbf{g} \text{ K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 800 \times 20^2 = 160\,000 \text{ J}$$

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 2

#### Question:

Find the change in potential energy of each of the following, stating in each case whether it is a loss or a gain:

- a a particle of mass 1.5 kg raised through a vertical distance of 3 m
- b a woman of mass 55 kg ascending a vertical distance of 15 m
- c a man of mass 75 kg descending a vertical distance of 30 m
- d a lift of mass 580 kg descending a vertical distance of 6 m
- e a man of mass 70 kg ascending a vertical distance of 36 m
- f a ball of mass 0.6 kg falling a vertical distance of 12 m
- g a lift of mass 800 kg ascending a vertical distance of 16 m

#### Solution:

a gain of P.E. =  $mgh = 1.5 \times 9.8 \times 3 = 44.1 \text{ J}$

b gain of P.E. =  $mgh = 55 \times 9.8 \times 15 = 8085 \text{ J}$

c loss of P.E. =  $mgh = 75 \times 9.8 \times 30 = 22\,050 \text{ J}$

d loss of P.E. =  $mgh = 580 \times 9.8 \times 6 = 34\,104 \text{ J}$

e gain of P.E. =  $mgh = 70 \times 9.8 \times 36 = 24\,696 \text{ J}$

f loss of P.E. =  $mgh = 0.6 \times 9.8 \times 12 = 70.56 = 70.6 \text{ J}$

g gain of P.E. =  $mgh = 800 \times 9.8 \times 16 = 125\,440 \text{ J}$

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 3

#### Question:

A particle of mass 1.2 kg decreases its speed from  $12 \text{ m s}^{-1}$  to  $4 \text{ m s}^{-1}$ . Calculate the decrease in the particle's kinetic energy.

#### Solution:

$$\begin{aligned}\text{Decrease in K.E.} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 1.2 \times 12^2 - \frac{1}{2} \times 1.2 \times 4^2 \\ &= 76.8\end{aligned}$$

The decrease in the particle's K.E. is 76.8 J.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 4

#### Question:

A van of mass 900 kg increases its speed from  $5 \text{ m s}^{-1}$  to  $20 \text{ m s}^{-1}$ . Calculate the increase in the van's kinetic energy.

#### Solution:

$$\begin{aligned}\text{Increase in K.E.} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 900 \times 20^2 - \frac{1}{2} \times 900 \times 5^2 \\ &= 168750\end{aligned}$$

The increase in the van's K.E. is 168 750 J.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 5

#### Question:

A particle of mass 0.2 kg increases its speed from  $2 \text{ m s}^{-1}$  to  $v \text{ m s}^{-1}$ . The particle's kinetic energy increases by 6 J. Calculate the value of  $v$ .

#### Solution:

$$\begin{aligned}\text{Increase in K.E.} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ 6 &= \frac{1}{2} \times 0.2 \times v^2 - \frac{1}{2} \times 0.2 \times 2^2 \\ 6 &= 0.1v^2 - 0.4 \\ v^2 &= \frac{6.4}{0.1} = 64 \\ v &= 8 \quad (v > 0)\end{aligned}$$

The value of  $v$  is 8.

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## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 6

#### Question:

An ice-skater of mass 45 kg is initially moving at  $5 \text{ m s}^{-1}$ . She decreases her kinetic energy by 100 J. Calculate her final speed.

#### Solution:

$$\begin{aligned}\text{Decrease in K.E.} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 && \leftarrow \text{Speed is positive.} \\ 100 &= \frac{1}{2} \times 45 \times 5^2 - \frac{1}{2} \times 45v^2 \\ 100 &= 562.5 - 22.5v^2 \\ v^2 &= \frac{462.5}{22.5} \\ v &= \pm 4.533 \\ v &= 4.533 \quad (v > 0)\end{aligned}$$

The skater's final speed is  $4.53 \text{ m s}^{-1}$ .

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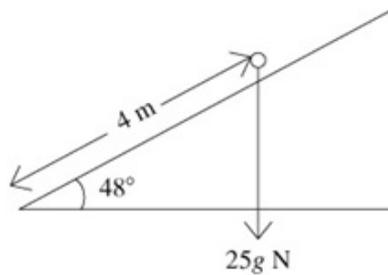
## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 7

#### Question:

A playground slide is a plane inclined at  $48^\circ$  to the horizontal. A child of mass 25 kg slides down the slide for 4 m. Calculate the potential energy lost by the child.

#### Solution:



$$\begin{aligned}\text{P.E. lost} &= mgh \\ &= 25 \times 9.8 \times (4 \sin 48^\circ) \\ &= 728.2\end{aligned}$$

← Vertical distance moved is  $4 \sin 48^\circ$ .

The P.E. lost by the child is 728 J.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 8

#### Question:

A ball of mass 0.6 kg is dropped from a height of 2 m into a pond.

**a** Calculate the kinetic energy of the ball as it hits the surface of the water.

The ball begins to sink in the water with a speed of  $4.8 \text{ m s}^{-1}$ .

**b** Calculate the kinetic energy lost when the ball strikes the water.

#### Solution:

**a**

$$s = 2 \text{ m}$$

$$a = 9.8 \text{ m s}^{-2}$$

$$u = 0$$

$$v = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 9.8 \times 2$$

$$v^2 = 39.2$$

$$\begin{aligned} \text{K.E.} &= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.6 \times 39.2 \\ &= 11.76 \end{aligned}$$

The K.E. of the ball as it hits the surface of the water is 11.8 J.

**b**

$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= 11.76 - \frac{1}{2} \times 0.6 \times 4.8^2 \\ &= 4.848 \end{aligned}$$

The K.E. lost by the ball is 4.85 J.

Use  $v^2 = u^2 + 2as$  to find the speed of the ball as it hits the water.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 9

#### Question:

A lorry of mass 2000 kg is initially travelling at  $35 \text{ m s}^{-1}$ . The brakes are applied, causing the lorry to decelerate at  $1.2 \text{ m s}^{-2}$  for 5 s. Calculate the loss of kinetic energy of the lorry.

#### Solution:

$$\begin{aligned}u &= 35 \text{ m s}^{-1} \\a &= -1.2 \text{ m s}^{-2} \\t &= 5 \text{ s} \\v &= ?\end{aligned}$$

$$\begin{aligned}v &= u + at \\v &= 35 - 1.2 \times 5 \\v &= 29\end{aligned}$$

Use  $v = u + at$  to find the final speed of the lorry.

$$\begin{aligned}\text{Loss of K.E.} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\&= \frac{1}{2} \times 2000 \times 35^2 - \frac{1}{2} \times 2000 \times 29^2 \\&= 384\,000\end{aligned}$$

The loss of K.E. of the lorry is 384 000 J.

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## Edexcel AS and A Level Modular Mathematics

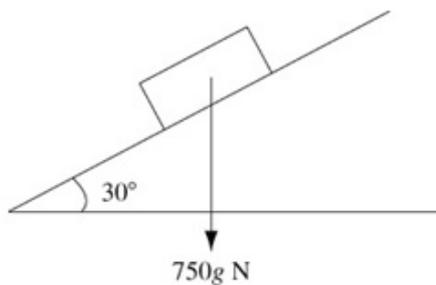
### Exercise B, Question 10

#### Question:

A car of mass 750 kg moves along a stretch of road which can be modelled as a line of greatest slope of a plane inclined to the horizontal at  $30^\circ$ . As the car moves up the road for 500 m its speed reduces from  $20 \text{ m s}^{-1}$  to  $15 \text{ m s}^{-1}$ . Calculate

- the loss of kinetic energy of the car,
- the gain of potential energy of the car.

#### Solution:



**a**

$$\begin{aligned} \text{Loss of K.E.} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 750 \times 20^2 - \frac{1}{2} \times 750 \times 15^2 \\ &= 65\,625 \end{aligned}$$

The loss of K.E. of the car is 65 625 J.

**b**

$$\begin{aligned} \text{Gain of P.E.} &= mgh \\ &= 750 \times 9.8 \times (500 \sin 30^\circ) \\ &= 1\,837\,500 \end{aligned}$$

The gain of P.E. of the car is 1 837 500 J.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 11

#### Question:

A man of mass 80 kg climbs a vertical cliff face of height  $h$  m. His potential energy increases by 15.7 kJ. Calculate the height of the cliff.

#### Solution:

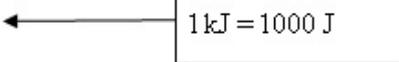
Increase of P.E. =  $mgh$

$$15.7 \times 1000 = 80 \times 9.8 h$$

$$h = \frac{15.7 \times 1000}{80 \times 9.8}$$

$$h = 20.02$$

The cliff is 20.0 m high.


$$1 \text{ kJ} = 1000 \text{ J}$$

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 1

#### Question:

Whenever a numerical value of  $g$  is required take  $g = 9.8 \text{ m s}^{-2}$ .

A particle of mass  $0.4 \text{ kg}$  falls a vertical distance of  $7 \text{ m}$  from rest.

- Calculate the potential energy lost.
- By assuming that air resistance can be neglected, calculate the final speed of the particle.

#### Solution:

**a**

$$\begin{aligned} \text{P.E. lost} &= mgh = 0.4 \times 9.8 \times 7 \\ &= 27.44 \end{aligned}$$

The P.E. lost is  $27.4 \text{ J}$ .

**b**

$$\begin{aligned} \text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 0.4 \times v^2 - 0 \end{aligned}$$

P.E. lost = K.E. gained

$$27.44 = \frac{1}{2} \times 0.4 \times v^2$$

$$v^2 = \frac{27.44}{0.2}$$

$$v = 11.71$$

The final speed of the particle is  $11.7 \text{ m s}^{-1}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 2

#### Question:

A stone of mass 0.5 kg is dropped from the top of a tower and falls vertically to the ground. It hits the ground with a speed of  $12 \text{ m s}^{-1}$ . Find

- the kinetic energy gained by the stone,
- the potential energy lost by the stone,
- the height of the tower.

#### Solution:

**a**

$$\begin{aligned} \text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 0.5 \times 12^2 - 0 \\ &= 36 \end{aligned}$$

The K.E. gained by the stone is 36 J.

**b**

$$\begin{aligned} \text{P.E. lost} &= \text{K.E. gained} \\ &= 36 \text{ J} \end{aligned}$$

The P.E. lost by the stone is 36 J.

P.E. lost = K.E. gained



**c**

$$\begin{aligned} \text{P.E. lost} &= mgh \\ 36 &= 0.5 \times 9.8 \times h \\ h &= \frac{36}{0.5 \times 9.8} \\ h &= 7.346 \end{aligned}$$

The height of the tower is 7.35 m.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 3

#### Question:

A box of mass 6 kg is pulled in a straight line across a smooth horizontal floor by a constant horizontal force of magnitude 10 N. The box has speed  $2.5 \text{ m s}^{-1}$  when it passes through point  $P$  and speed  $5 \text{ m s}^{-1}$  when it passes through point  $Q$ .

- Find the increase in kinetic energy of the box.
- Write down the work done by the force.
- Find the distance  $PQ$ .

#### Solution:



#### a

$$\begin{aligned} \text{Increase in K.E.} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 6 \times 5^2 - \frac{1}{2} \times 6 \times 2.5^2 \\ &= 56.25 \end{aligned}$$

The increase in K.E. of the box is 56.3 J.

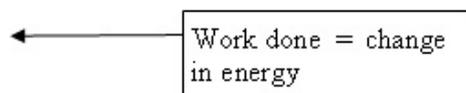
- The work done by the force is 56.3 J.

#### c

$$\begin{aligned} \text{Work done} &= F \times s \\ 56.25 &= 10 \times s \end{aligned}$$

$$s = \frac{56.25}{10} = 5.625$$

The distance  $PQ$  is 5.63 m.



# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

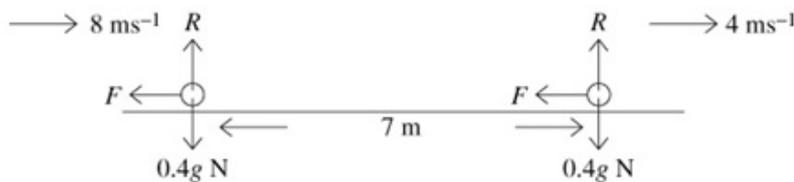
### Exercise C, Question 4

#### Question:

A particle of mass  $0.4 \text{ kg}$  moves in a straight line across a rough horizontal surface. The speed of the particle decreases from  $8 \text{ m s}^{-1}$  to  $4 \text{ m s}^{-1}$  as it travels  $7 \text{ m}$ .

- Calculate the kinetic energy lost by the particle.
- Write down the work done against friction.
- Calculate the coefficient of friction between the particle and the surface.

#### Solution:



**a**

$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 0.4 \times 8^2 - \frac{1}{2} \times 0.4 \times 4^2 \\ &= 9.6 \end{aligned}$$

The K.E. lost by the particle is  $9.6 \text{ J}$ .

**b** The work done against friction is  $9.6 \text{ J}$ .

Work done = change  
in energy

**c**

$$R(\uparrow) \quad R = 0.4g$$

$$\text{Friction is limiting} \quad F = \mu R$$

$$F = 0.4g \times \mu$$

$$\text{Work done} = F \times s$$

$$9.6 = 0.4g \times \mu \times 7$$

$$\mu = \frac{9.6}{0.4 \times 9.8 \times 7} = 0.3498$$

The coefficient of friction is  $0.350$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

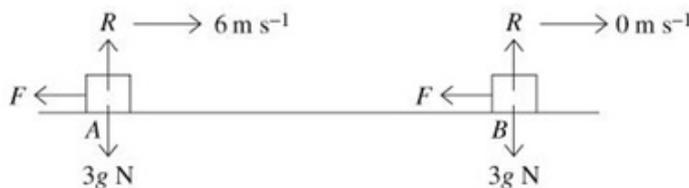
### Exercise C, Question 5

#### Question:

A box of mass 3 kg is projected from point  $A$  of a rough horizontal floor with speed  $6 \text{ m s}^{-1}$ . The box moves in a straight line across the floor and comes to rest at point  $B$ . The coefficient of friction between the box and the floor is 0.4.

- Calculate the kinetic energy lost by the box.
- Write down the work done against friction.
- Calculate the distance  $AB$ .

#### Solution:



**a**

$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 3 \times 6^2 - 0 \\ &= 54 \end{aligned}$$

$$\mu = 0.4$$

The kinetic energy lost by the box is 54 J.

**b** The work done against friction is 54 J.

**c**

$$R(\uparrow) \quad R = 3g$$

$$\text{Friction is limiting: } F = \mu R$$

$$F = 0.4 \times 3g$$

$$\text{Work done} = F \times s$$

$$54 = 0.4 \times 3g \times s$$

$$s = \frac{54}{0.4 \times 3g} = 4.591$$

The distance  $AB$  is 4.59 m.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 6

#### Question:

A particle of mass 0.8 kg falls a vertical distance of 5 m from rest. By considering energy, find the speed of the particle as it hits the ground. (You may assume that air resistance can be neglected.)

#### Solution:

$$\begin{aligned}\text{P.E. lost} &= mgh \\ &= 0.8 \times 9.8 \times 5 \\ &= 39.2\end{aligned}$$

$$\begin{aligned}\text{K.E. gained} &= \text{P.E. lost} \\ &= 39.2\end{aligned}$$

$$\begin{aligned}\text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ 39.2 &= \frac{1}{2} \times 0.8v^2 - 0 \\ v^2 &= \frac{39.2 \times 2}{0.8} \\ v &= 9.899\end{aligned}$$

The particle hits the ground at  $9.90 \text{ m s}^{-1}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 7

#### Question:

A stone of mass 0.3 kg is dropped from the top of a vertical cliff and falls freely under gravity. It hits the ground below with a speed of  $20 \text{ m s}^{-1}$ . Use energy considerations to calculate the height of the cliff.

#### Solution:

$$\begin{aligned}\text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 0.3 \times 20^2 - 0 \\ &= 60\end{aligned}$$

$$\begin{aligned}\text{P.E. lost} &= \text{K.E. gained} \\ &= 60\end{aligned}$$

$$\begin{aligned}\text{P.E. lost} &= mgh \\ 60 &= 0.3 \times 9.8 \times h \\ h &= \frac{60}{0.3 \times 9.8} \\ h &= 20.40\end{aligned}$$

The cliff is 20.4 m high.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 8

#### Question:

A particle of mass 0.3 kg is projected vertically upwards and moves freely under gravity. The initial speed of the particle is  $u \text{ m s}^{-1}$ . When the particle is 5 m above the point of projection its kinetic energy is 2.1 J. Calculate the value of  $u$ .

#### Solution:

$$\begin{aligned}\text{P.E. gained} &= mgh \\ &= 0.3 \times 9.8 \times 5\end{aligned}$$

$$\text{K.E. lost} = \text{initial K.E.} - \text{final K.E.}$$

$$= \frac{1}{2}mu^2 - 2.1$$

$$= \frac{1}{2} \times 0.3u^2 - 2.1$$

$$\text{K.E. lost} = \text{P.E. gained}$$

$$\frac{1}{2} \times 0.3u^2 - 2.1 = 0.3 \times 9.8 \times 5$$

$$u^2 = \frac{0.3 \times 9.8 \times 5 + 2.1}{\frac{1}{2} \times 0.3}$$

$$u = 10.58$$

$$u = 10.6$$

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 9

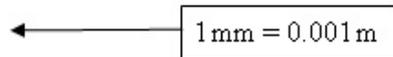
#### Question:

A bullet of mass 0.1 kg travelling at  $500 \text{ m s}^{-1}$  horizontally hits a vertical wall. The bullet penetrates the wall to a depth of 50 mm. The resistive force exerted on the bullet by the wall is constant. Calculate the magnitude of the resistive force.

#### Solution:

$$\begin{aligned} \text{Loss of K.E.} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 0.1 \times 500^2 - 0 \end{aligned}$$

$$\begin{aligned} \text{Work done by resistance} &= F \times s \\ &= F \times 0.05 \end{aligned}$$



$$1 \text{ mm} = 0.001 \text{ m}$$

Work done by resistance = Loss of K.E.

$$\begin{aligned} F \times 0.05 &= \frac{1}{2} \times 0.1 \times 500^2 \\ F &= \frac{\frac{1}{2} \times 0.1 \times 500^2}{0.05} \\ &= 250\,000 \end{aligned}$$

The magnitude of the resistive force is 250 000 N (or 250 kN).

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 10

#### Question:

A bullet of mass 150 g travelling at  $500 \text{ m s}^{-1}$  horizontally hits a vertical wall. The wall exerts a constant resistance of magnitude 250 000 N on the bullet. Calculate the distance the bullet penetrates the wall.

#### Solution:

$$\begin{aligned} \text{Loss of K.E.} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 0.15 \times 500^2 - 0 \end{aligned}$$

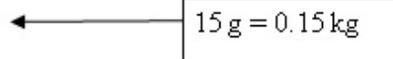
$$\begin{aligned} \text{Work done by resistance} &= F \times s \\ &= 250\,000 s \end{aligned}$$

$$\text{Work done by resistance} = \text{Loss of K.E.}$$

$$250\,000 s = \frac{1}{2} \times 0.15 \times 500^2$$

$$s = \frac{\frac{1}{2} \times 0.15 \times 500^2}{250\,000}$$

$$= 0.075$$



$$15 \text{ g} = 0.15 \text{ kg}$$

The distance the bullet penetrates the wall is 0.075 m (or 75 mm).

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

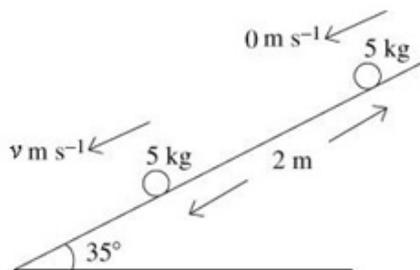
### Exercise C, Question 11

#### Question:

A package of mass 5 kg is released from rest and slides 2 m down a line of greatest slope of a smooth plane inclined at  $35^\circ$  to the horizontal.

- Calculate the potential energy lost by the package.
- Write down the kinetic energy gained by the package.
- Calculate the final speed of the package.

#### Solution:



**a**

$$\begin{aligned} \text{P.E. lost} &= mgh \\ &= 5 \times 9.8 \times (2 \sin 35^\circ) \\ &= 56.21 \end{aligned}$$

The P.E. lost is 56.2 J.

**b**

The K.E. gained is 56.2 J.

**c**

$$\begin{aligned} \text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ 56.2 &= \frac{1}{2} \times 5 \times v^2 - 0 \\ v^2 &= \frac{56.2 \times 2}{5} \\ v &= 4.741 \end{aligned}$$

The final speed of the package is  $4.74 \text{ m s}^{-1}$ .

# Solutionbank M2

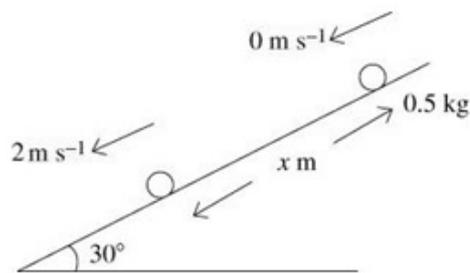
## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 12

#### Question:

A particle of mass  $0.5 \text{ kg}$  is released from rest and slides down a line of greatest slope of a smooth plane inclined at  $30^\circ$  to the horizontal. When the particle has moved a distance  $x \text{ m}$ , its speed is  $2 \text{ m s}^{-1}$ . Find the value of  $x$ .

#### Solution:



$$\begin{aligned} \text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 0.5 \times 2^2 - 0 \\ &= 1 \end{aligned}$$

$$\text{P.E. lost} = mgh = 0.5 \times 9.8 \times (x \sin 30^\circ)$$

$$\text{P.E. lost} = \text{K.E. gained}$$

$$0.5 \times 9.8 \times (x \sin 30^\circ) = 1$$

$$\begin{aligned} x &= \frac{1}{0.5 \times 9.8 \times \sin 30^\circ} \\ &= 0.4081 \\ x &= 0.408 \end{aligned}$$

# Solutionbank M2

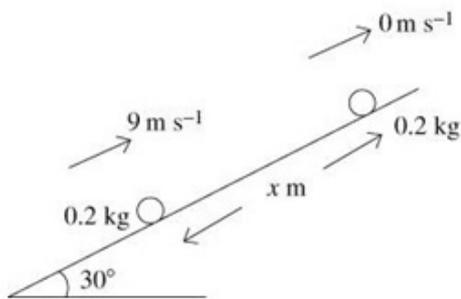
## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 13

#### Question:

A particle of mass  $0.2 \text{ kg}$  is projected with speed  $9 \text{ m s}^{-1}$  up a line of greatest slope of a smooth plane inclined at  $30^\circ$  to the horizontal. The particle travels a distance  $x \text{ m}$  before first coming to rest. By considering energy, calculate the value of  $x$ .

#### Solution:



$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 0.2 \times 9^2 - 0 \end{aligned}$$

$$\begin{aligned} \text{P.E. gained} &= mgh \\ &= 0.2 \times 9.8(x \sin 30^\circ) \end{aligned}$$

$$\text{P.E. gained} = \text{K.E. lost}$$

$$0.2 \times 9.8(x \sin 30^\circ) = \frac{1}{2} \times 0.2 \times 9^2$$

$$\begin{aligned} x &= \frac{\frac{1}{2} \times 0.2 \times 9^2}{0.2 \times 9.8 \sin 30^\circ} \\ &= 8.265 \\ &= 8.27 \end{aligned}$$

# Solutionbank M2

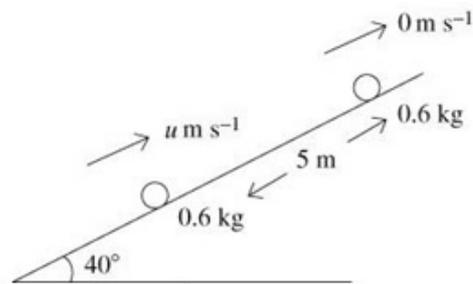
## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 14

#### Question:

A particle of mass 0.6 kg is projected up a line of greatest slope of a smooth plane inclined at  $40^\circ$  to the horizontal. The particle travels 5 m before first coming to rest. Use energy considerations to calculate the speed of projection.

#### Solution:



$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 0.6u^2 - 0 \end{aligned}$$

$$\begin{aligned} \text{P.E. gained} &= mgh \\ &= 0.6 \times 9.8 \times 5 \sin 40^\circ \end{aligned}$$

$$\text{K.E. lost} = \text{P.E. gained}$$

$$\frac{1}{2} \times 0.6u^2 = 0.6 \times 9.8 \times 5 \sin 40^\circ$$

$$u^2 = \frac{0.6 \times 9.8 \times 5 \sin 40^\circ}{\frac{1}{2} \times 0.6}$$

$$u = 7.936$$

The speed of projection is  $7.94 \text{ m s}^{-1}$ .

## Solutionbank M2

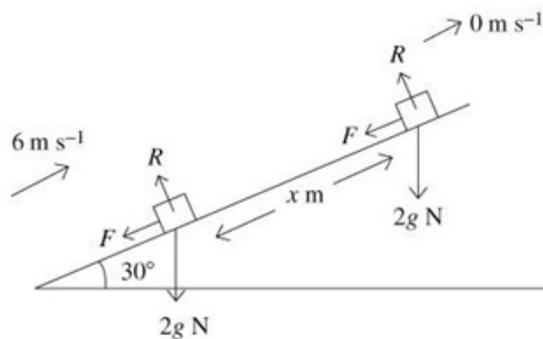
### Edexcel AS and A Level Modular Mathematics

#### Exercise C, Question 15

#### Question:

A box of mass 2 kg is projected with speed  $6 \text{ m s}^{-1}$  up a line of greatest slope of a rough plane inclined at  $30^\circ$  to the horizontal. The coefficient of friction between the box and the plane is  $\frac{1}{3}$ . Use the work-energy principle to calculate the distance the box travels up the plane before first coming to rest.

#### Solution:



$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 2 \times 6^2 - 0 \\ &= 36 \end{aligned}$$

$$\begin{aligned} \text{P.E. gained} &= mgh \\ &= 2 \times 9.8(x \sin 30^\circ) \\ &= 9.8x \end{aligned}$$

$$R(\sphericalangle)R = 2g \sin 30^\circ$$

Friction is limiting:  $F = \mu R$

$$F = \frac{1}{3} \times 2g \sin 30^\circ = \frac{1}{3}g$$

Work done against friction =  $Fx$

$$= \frac{1}{3}gx$$

K.E. lost = P.E. gained + work done against  $F$

$$36 = 9.8 \times x + \frac{1}{3} \times 9.8 \times x$$

$$36 = \frac{4}{3} \times 9.8 \times x$$

$$x = \frac{36 \times 3}{4 \times 9.8}$$

$$x = 2.755$$

The particle moves 2.76 m up the plane.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

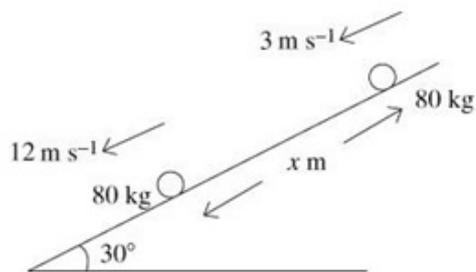
### Exercise C, Question 16

#### Question:

A cyclist freewheels down a hill inclined at  $30^\circ$  to the horizontal. The cyclist and his cycle have a combined mass of 80 kg. His speed increases from  $3 \text{ m s}^{-1}$  to  $12 \text{ m s}^{-1}$ . Assuming that resistances can be ignored, calculate

- the potential energy lost by the cyclist,
- the distance travelled by the cyclist.

#### Solution:



**a**

$$\begin{aligned} \text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 80 \times 12^2 - \frac{1}{2} \times 80 \times 3^2 \\ &= 5400 \end{aligned}$$

$$\text{P.E. lost} = \text{K.E. gained}$$

$\therefore$  The P.E. lost is 5400 J.

P.E. lost = K.E. gained

**b**

$$\text{P.E. lost} = mgh$$

$$5400 = 80 \times 9.8 \times (x \sin 30^\circ)$$

$$\begin{aligned} x &= \frac{5400}{80 \times 9.8 \times \sin 30^\circ} \\ &= 13.77 \end{aligned}$$

The cyclist travels 13.8 m.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 17

#### Question:

A cyclist starts from rest and freewheels down a hill inclined at  $20^\circ$  to the horizontal. After travelling 60 m the road becomes horizontal and the cyclist travels a further 50 m before coming to rest. The cyclist and her cycle have a combined mass of 70 kg and the resistance to motion remains constant throughout. Calculate the magnitude of the resistance.

#### Solution:



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# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 1

#### Question:

Whenever a numerical value of  $g$  is required take  $g = 9.8 \text{ m s}^{-2}$ .

A force of 1500 N pulls a van up a slope at a constant speed of  $12 \text{ m s}^{-1}$ . Calculate, in kW, the power developed.

#### Solution:

$$\begin{aligned}\text{Power} &= F \times v \\ &= 1500 \times 12 \\ &= 18\,000\end{aligned}$$

The power is 18 kW.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 2

#### Question:

A car is travelling at  $15 \text{ m s}^{-1}$  and its engine is producing a driving force of  $1000 \text{ N}$ .  
Calculate the power developed.

#### Solution:

$$\begin{aligned}\text{Power} &= F \times v \\ &= 1000 \times 15 \\ &= 15\,000\end{aligned}$$

The power is  $15\,000 \text{ W}$  (or  $15 \text{ kW}$ ).

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 3

#### Question:

The engine of a van is working at 5 kW and the van is travelling at  $18 \text{ m s}^{-1}$ . Find the magnitude of the driving force produced by the van's engine.

#### Solution:

$$\text{Power} = F \times v$$

$$5000 = F \times 18$$

$$F = \frac{5000}{18}$$

$$= 277.7$$

The driving force has magnitude 278 N.

# Solutionbank M2

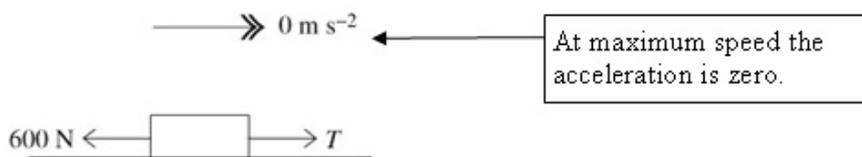
## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 4

#### Question:

A car's engine is working at 15 kW. The car is travelling along a horizontal road. The total resistance to motion has a magnitude of 600 N. Calculate the maximum speed of the car.

#### Solution:



$$R \left( \rightarrow \right) T = 600$$

$$\text{Power} = F \times v$$

$$15000 = 600v$$

$$v = \frac{15000}{600}$$

$$= 25$$

The maximum speed is  $25 \text{ m s}^{-1}$ .

# Solutionbank M2

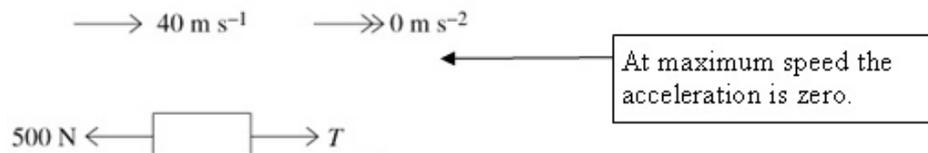
## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 5

#### Question:

A car has a maximum speed of  $40 \text{ m s}^{-1}$  when travelling along a horizontal road against a constant resistance of  $500 \text{ N}$ . Calculate the power the car's engine must develop to maintain this speed.

#### Solution:



$$R(\rightarrow) T = 500$$

$$\begin{aligned} \text{Power} &= F \times v \\ &= 500 \times 40 \\ &= 20\,000 \end{aligned}$$

The power is  $20\,000 \text{ W}$  (or  $20 \text{ kW}$ ).

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 6

#### Question:

A van is travelling along a horizontal road at a constant speed of  $16 \text{ m s}^{-1}$ . The van's engine is working at  $8.8 \text{ kW}$ . Calculate the magnitude of the resistance to motion.

#### Solution:

$$\longrightarrow 16 \text{ m s}^{-1} \quad \longrightarrow\!\!\!> 0 \text{ m s}^{-2}$$



$$\text{Power} = F \times v$$

$$8.8 \times 10^3 = T \times 16$$

$$T = \frac{8800}{16}$$

$$T = 550$$

$$R(\rightarrow) R = T$$

$$R = 550$$

The magnitude of the resistance is  $550 \text{ N}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 7

#### Question:

A car of mass 850 kg is travelling along a straight horizontal road against resistances totalling 350 N. The car's engine is working at 9 kW. Calculate

- a the acceleration when the car is travelling at  $7 \text{ m s}^{-1}$ ,
- b the acceleration when the car is travelling at  $15 \text{ m s}^{-1}$ ,
- c the maximum speed of the car.

#### Solution:

**a**

$$\longrightarrow 7 \text{ m s}^{-1} \quad \longrightarrow\!\!\! \longrightarrow a \text{ m s}^{-2}$$

$$350 \text{ N} \longleftarrow \boxed{850 \text{ kg}} \longrightarrow T$$

$$\text{Power} = F \times v$$

$$9000 = T \times 7$$

$$T = \frac{9000}{7}$$

$$F = ma$$

$$\frac{9000}{7} - 350 = 850a$$

$$a = \frac{\frac{9000}{7} - 350}{850}$$

$$a = 1.100$$

The acceleration is  $1.10 \text{ m s}^{-2}$ .

First find the fraction of the force produced by the engine and then use  $F = ma$  to find the acceleration.

**b**

$$\longrightarrow 15 \text{ m s}^{-1} \quad \longrightarrow\!\!\! \longrightarrow a \text{ m s}^{-2}$$

$$350 \text{ N} \longleftarrow \boxed{850 \text{ kg}} \longrightarrow T$$

$$\text{Power} = F \times v$$

$$9000 = T \times 15$$

$$T = \frac{9000}{15} = 600$$

$$F = ma$$

$$600 - 350 = 850a$$

$$a = \frac{250}{850}$$

$$a = 0.2941$$

The acceleration is  $0.294 \text{ m s}^{-2}$ .

**c**

$$\longrightarrow 0 \text{ m s}^{-1} \quad \longrightarrow\!\!\! \longrightarrow 0 \text{ m s}^{-2}$$

$$350 \text{ N} \longleftarrow \boxed{850 \text{ kg}} \longrightarrow T$$

$$R(\rightarrow)T = 350$$

$$\text{Power} = F \times v$$

$$9000 = 350v$$

$$v = \frac{9000}{350}$$

$$v = 25.71$$

The maximum speed is  $25.7 \text{ m s}^{-1}$ .



# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

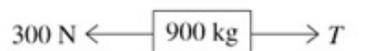
### Exercise D, Question 8

#### Question:

A car of mass 900 kg is travelling along a straight horizontal road at a speed of  $20 \text{ m s}^{-1}$ . The constant resistances to motion total 300 N. The car is accelerating at  $0.3 \text{ m s}^{-2}$ . Calculate the power developed by the engine.

#### Solution:

$$\longrightarrow 20 \text{ m s}^{-1} \quad \longrightarrow\!\!\!> 0.3 \text{ m s}^{-2}$$



$$F = ma$$

$$T - 300 = 900 \times 0.3$$

$$T = 900 \times 0.3 + 300$$

$$= 570$$

$$\text{Power} = F \times v$$

$$= 570 \times 20$$

$$= 1140$$

The power developed by the engine is 1140 W (or 1.14 kW).

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 9

#### Question:

A car of mass 1000 kg is travelling along a straight horizontal road. The car's engine is working at 12 kW. When its speed is  $24 \text{ m s}^{-1}$  its acceleration is  $0.2 \text{ m s}^{-2}$ . The resistances to motion have a total magnitude of  $R$  newtons. Calculate the value of  $R$ .

#### Solution:

$$\longrightarrow 24 \text{ m s}^{-1} \quad \longrightarrow\!\!\! \gg 0.2 \text{ m s}^{-2}$$



$$\text{Power} = F \times v$$

$$12000 = T \times 24$$

$$T = \frac{12000}{24} = 500$$

$$F = ma$$

$$T - R = 1000 \times 0.2$$

$$500 - R = 200$$

$$R = 500 - 200$$

$$R = 300$$

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 10

#### Question:

A cyclist is travelling along a straight horizontal road. The resistance to his motion is constant and has magnitude 28 N. The maximum rate at which he can work is 280 W. Calculate his maximum speed.

#### Solution:

$$\longrightarrow v \text{ m s}^{-1} \longrightarrow \gg 0 \text{ m s}^{-2}$$



$$R(\rightarrow) T = 28$$

$$\text{Power} = F \times v$$

$$280 = 28v$$

$$v = 10$$

His maximum speed is  $10 \text{ m s}^{-1}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 11

#### Question:

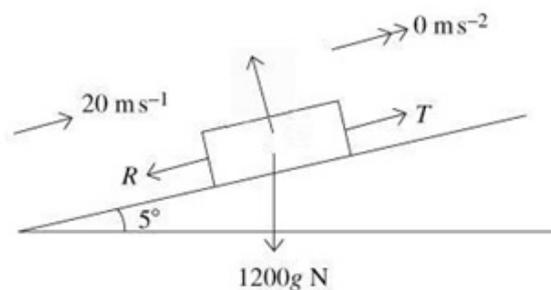
A van of mass 1200 kg is travelling up a straight road inclined at  $5^\circ$  to the horizontal. The van moves at a constant speed of  $20 \text{ m s}^{-1}$  and its engine is working at 24 kW. The resistance to motion from non-gravitational forces has magnitude  $R$  newtons.

**a** Calculate the value of  $R$ .

The road now becomes horizontal. The resistance to motion from non-gravitational forces is unchanged.

**b** Calculate the initial acceleration of the car.

#### Solution:

**a**

$$\text{Power} = F \times v$$

$$24000 = T \times 20$$

$$T = \frac{24000}{20} = 1200$$

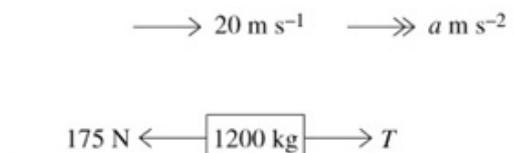
$$R(\nearrow) \quad T = R + 1200g \sin 5^\circ$$

$$1200 = R + 1200g \sin 5^\circ$$

$$R = 1200 - 1200g \sin 5^\circ$$

$$R = 175.0$$

The magnitude of the resistance is 175 N.

**b**

From above,  $T = 1200$ .

$$F = ma$$

$$1200 - 175 = 1200a$$

$$a = \frac{1200 - 175}{1200}$$

$$a = 0.8541$$

The initial acceleration is  $0.854 \text{ m s}^{-2}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 12

#### Question:

A car of mass  $800 \text{ kg}$  is travelling at  $18 \text{ m s}^{-1}$  along a straight horizontal road. The car's engine is working at a constant rate of  $26 \text{ kW}$  against a constant resistance of magnitude  $750 \text{ N}$ .

**a** Find the acceleration of the car.

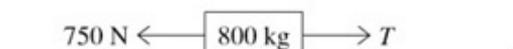
The car now ascends a straight road, inclined at  $9^\circ$  to the horizontal. The resistance to motion from non-gravitational forces is unchanged and the car's engine works at the same rate.

**b** Find the maximum speed at which the car can travel up the road.

#### Solution:

a

$$\longrightarrow 18 \text{ m s}^{-1} \quad \longrightarrow a \text{ m s}^{-2}$$



$$\text{Power} = F \times v$$

$$26000 = T \times 18$$

$$T = \frac{26000}{18}$$

$$F = ma$$

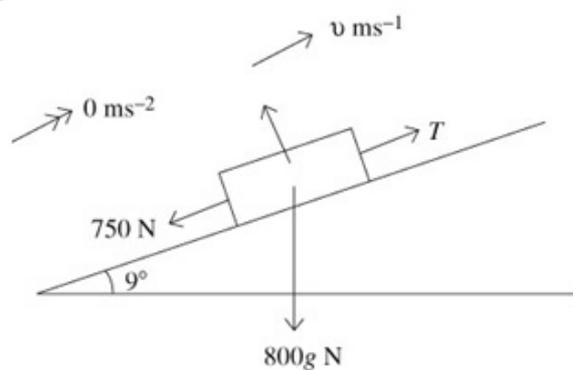
$$T - 750 = 800a$$

$$800a = \frac{26000}{18} - 750$$

$$a = 0.8680$$

The acceleration is  $0.868 \text{ m s}^{-2}$ .

b



$$R(\wedge) T = 750 + 800g \sin 9^\circ$$

$$\text{Power} = F \times v$$

$$26000 = T \times v$$

$$26000 = (750 + 800 \times 9.8 \sin 9^\circ)v$$

$$v = \frac{26000}{(750 + 800 \times 9.8 \sin 9^\circ)}$$

$$v = 13.15$$

The maximum speed is  $13.2 \text{ m s}^{-1}$ .

## Solutionbank M2

### Edexcel AS and A Level Modular Mathematics

#### Exercise D, Question 13

#### Question:

A van of mass  $1500 \text{ kg}$  is travelling at its maximum speed of  $30 \text{ m s}^{-1}$  along a straight horizontal road against a constant resistance of magnitude  $600 \text{ N}$ .

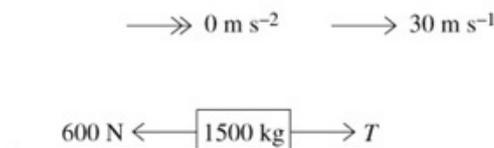
a Find the power developed by the van's engine.

The van now travels up a straight road inclined at  $8^\circ$  to the horizontal. The van's engine works at the same rate and the resistance to motion from non-gravitational forces is unchanged.

b Find the maximum speed at which the van can ascend the road.

#### Solution:

a

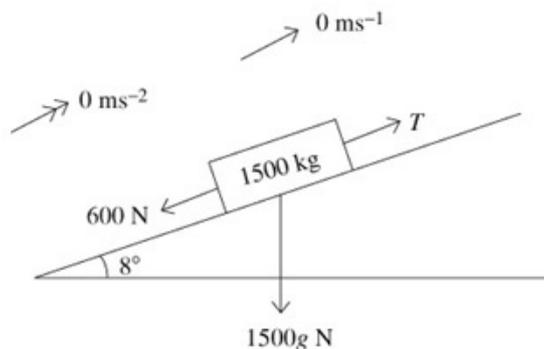


$$R(\rightarrow) T = 600$$

$$\begin{aligned} \text{Power} &= F \times v \\ &= 600 \times 30 \\ &= 18\,000 \end{aligned}$$

The power is  $18\,000 \text{ W}$  (or  $18 \text{ kW}$ ).

b



$$R(\rightarrow) T = 600 + 1500g \sin 8^\circ$$

$$\text{Power} = F \times v$$

$$18\,000 = (600 + 1500g \sin 8^\circ)v$$

$$v = \frac{18\,000}{(600 + 1500g \sin 8^\circ)} = 6.803$$

The maximum speed is  $6.80 \text{ m s}^{-1}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 14

#### Question:

A car is moving along a straight horizontal road with speed  $v \text{ m s}^{-1}$ . The magnitude of the resistance to motion of the car is given by the formula  $(150 + 3v) \text{ N}$ . The car's engine is working at  $10 \text{ kW}$ . Calculate the maximum value of  $v$ .

#### Solution:

$$\longrightarrow 0 \text{ m s}^{-2} \quad \longrightarrow v \text{ m s}^{-1}$$



$$R(\rightarrow) \quad T = 150 + 3v$$

$$\text{Power} = F \times v$$

$$10\,000 = (150 + 3v)v$$

$$3v^2 + 150v - 10\,000 = 0$$

$$v = \frac{-150 \pm \sqrt{(150^2 - 4 \times 3 \times (-10\,000))}}{2 \times 3}$$

$$= 37.91 \quad (v > 0)$$

The maximum value of  $v$  is  $37.9$

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 15

#### Question:

A train of mass 150 tonnes is moving up a straight track which is inclined at  $2^\circ$  to the horizontal. The resistance to the motion of the train from non-gravitational forces has magnitude 6 kN and the train's engine is working at a constant rate of 350 kW.

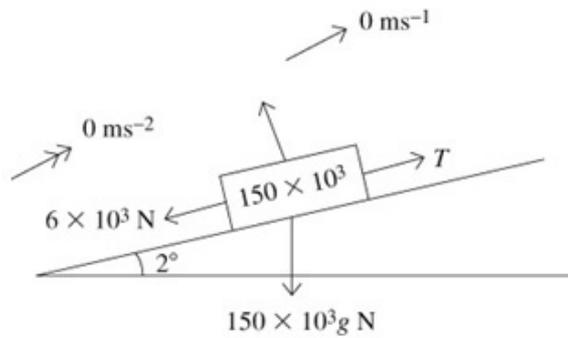
**a** Calculate the maximum speed of the train.

The track now becomes horizontal. The engine continues to work at 350 kW and the resistance to motion remains 6 kN.

**b** Find the initial acceleration of the train.

#### Solution:

a



$$R(\nearrow) T = 6 \times 10^3 + 150 \times 10^3 g \sin 2^\circ$$

$$\text{Power} = F \times v$$

$$350 \times 10^3 = (6 \times 10^3 + 150 \times 10^3 g \sin 2^\circ) \times v$$

$$v = \frac{350}{(6 + 150 \times 9.8 \sin 2^\circ)}$$

$$= 6.107$$

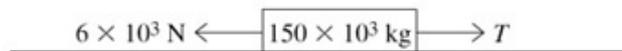
The maximum speed is  $6.11 \text{ m s}^{-1}$ .

1 tonne =  $10^3 \text{ kg}$

When tonnes, kilonewtons and kilowatts are used the  $10^3$  will cancel, leaving easier numbers.

b

$$\longrightarrow 6.107 \text{ m s}^{-1} \longrightarrow \gg a \text{ m s}^{-2}$$



$$\text{Power} = F \times v$$

$$350 \times 10^3 = T \times 6.107$$

$$T = \frac{350 \times 10^3}{6.107}$$

$$F = ma$$

$$T - 6 \times 10^3 = 150 \times 10^3 a$$

$$\frac{350 \times 10^3}{6.107} - 6 \times 10^3 = 150 \times 10^3 a$$

$$150a = \frac{350}{6.107} - 6$$

$$a = 0.3420$$

The initial acceleration is  $0.342 \text{ m s}^{-2}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

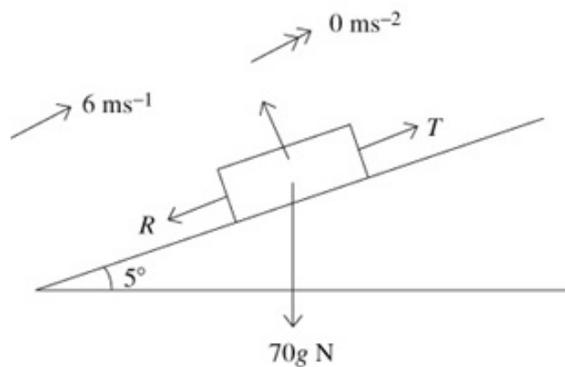
### Exercise E, Question 1

#### Question:

Whenever a numerical value of  $g$  is required take  $g = 9.8 \text{ m s}^{-2}$ .

A cyclist and her bicycle have a combined mass of  $70 \text{ kg}$ . She is cycling at a constant speed of  $6 \text{ m s}^{-1}$  on a straight road up a hill inclined at  $5^\circ$  to the horizontal. She is working at a constant rate of  $480 \text{ W}$ . Calculate the magnitude of the resistance to motion from non-gravitational forces.

#### Solution:



$$\text{Power} = F \times v$$

$$480 = T \times 6$$

$$T = \frac{480}{6} = 80$$

$$R(\rightarrow)T = R + 70g \sin 5^\circ$$

$$80 = R + 70 \times 9.8 \sin 5^\circ$$

$$R = 80 - 70 \times 9.8 \sin 5^\circ$$

$$R = 20.21$$

The magnitude of the resistance is  $20.2 \text{ N}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 2

#### Question:

A boy hauls a bucket of water through a vertical distance of 25 m. The combined mass of the bucket and water is 12 kg. The bucket starts from rest and finishes at rest.

**a** Calculate the work done by the boy.

The boy takes 30 s to raise the bucket.

**b** Calculate the average rate of working of the boy.

#### Solution:

**a**

$$\begin{aligned} \text{P.E. gained by water and bucket} &= mgh \\ &= 12 \times 9.8 \times 25 \\ &= 2940 \end{aligned}$$

$$\text{Initial K.E.} = \text{final K.E.} = 0$$

$$\begin{aligned} \text{Work done by the boy} &= \text{P.E. gained by bucket} \\ &= 2940 \text{ J} \end{aligned}$$

**b**

$$\begin{aligned} \text{Average rate of working} &= \frac{\text{work done}}{\text{time taken}} = \frac{2940}{30} \\ &= 98 \end{aligned}$$

$$\text{Rate of doing work} = \frac{\text{work done}}{\text{time taken}}$$

The average rate of working of the boy is  $98 \text{ J s}^{-1}$  (or 98 W).

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

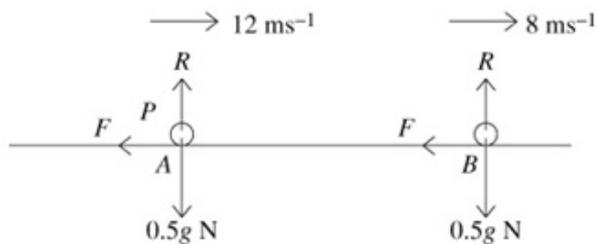
### Exercise E, Question 3

#### Question:

A particle  $P$  of mass  $0.5 \text{ kg}$  is moving in a straight line from  $A$  to  $B$  on a rough horizontal plane. At  $A$  the speed of  $P$  is  $12 \text{ m s}^{-1}$ , and at  $B$  its speed is  $8 \text{ m s}^{-1}$ . The distance from  $A$  to  $B$  is  $25 \text{ m}$ . The only resistance to motion is the friction between the particle and the plane. Find

- the work done by friction as  $P$  moves from  $A$  to  $B$ ,
- the coefficient of friction between the particle and the plane.

#### Solution:



**a**

$$\begin{aligned} \text{K.E. lost by particle} &= \frac{1}{2} \times 0.5 \times 12^2 - \frac{1}{2} \times 0.5 \times 8^2 \\ &= 20 \end{aligned}$$

Work done by friction = K.E. lost by particle

$\therefore$  Work done by friction =  $20 \text{ J}$

Work done by friction = K.E. lost by particle.

**b**

$$R(\uparrow)R = 0.5g$$

Friction is limiting  $F = \mu R = \mu \times 0.5g$

Work done by friction =  $F \times s$

$$20 = \mu \times 0.5g \times 25$$

$$\mu = \frac{20}{0.5g \times 25} = 0.1632$$

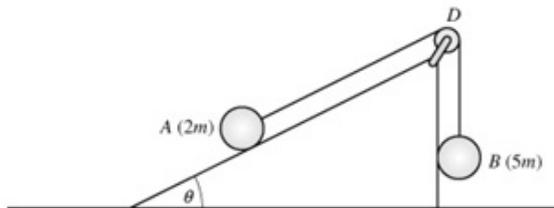
The coefficient of friction is  $0.163$ .

## Solutionbank M2

### Edexcel AS and A Level Modular Mathematics

#### Exercise E, Question 4

#### Question:



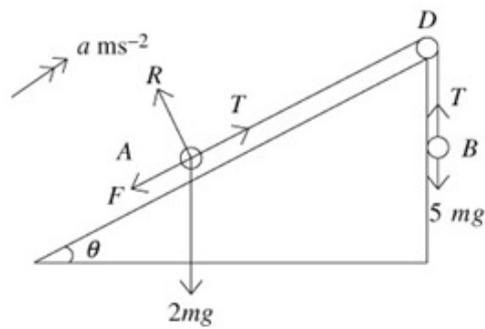
The diagram shows a particle  $A$  of mass  $2m$  which can move on the rough surface of a plane inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{3}{5}$ . A second particle  $B$  of mass  $5m$  hangs freely attached to a light inextensible string which passes over a smooth light pulley fixed at  $D$ . The other end of the string is attached to  $A$ . The coefficient of friction between  $A$  and the plane is  $\frac{3}{8}$ . Particle  $B$  is initially hanging 2 m above the ground and  $A$  is 4 m from  $D$ . When the system is released from rest with the string taut  $A$  moves up a line of greatest slope of the plane.

**a** Find the initial acceleration of  $A$ .

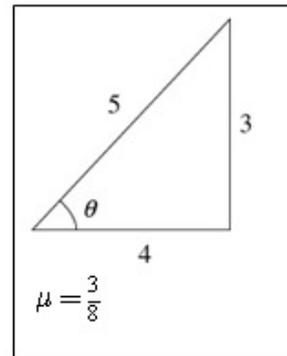
When  $B$  has descended 1 m the string breaks.

**b** By using the principle of conservation of energy calculate the total distance moved by  $A$  before it first comes to rest.

#### Solution:



$a \text{ ms}^{-2}$



**a**  $R(\perp)$  for  $A$ :

$$R = 2mg \cos \theta$$

Friction is limiting:

$$F = \mu R$$

$$F = \frac{3}{8} \times 2mg \cos \theta$$

$$= \frac{3}{8} \times 2mg \times \frac{4}{5}$$

$$= \frac{3}{5} mg$$

$$F = ma \text{ for } A: T - (F + 2mg \sin \theta) = 2ma$$

$$T - \left( \frac{3}{5} mg + 2mg \times \frac{3}{5} \right) = 2ma$$

$$T - \frac{9mg}{5} = 2ma \quad (1)$$

$$F = ma \text{ for } B: 5mg - T = 5ma \quad (2)$$

$$(1) + (2): \quad 5mg - \frac{9mg}{5} = 7ma$$

$$\frac{16mg}{5} = 7ma$$

$$a = \frac{16g}{35} = \frac{16 \times 9.8}{35}$$

$$a = 4.48$$

The acceleration of  $A$  is  $4.48 \text{ m s}^{-2}$ .

**b** For the first 1m  $A$  travels

$$u = 0$$

$$a = 4.48 \text{ m s}^{-2}$$

$$s = 1 \text{ m}$$

$$v = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = 2 \times 4.48 \times 1$$

$$v^2 = 8.96$$

After string breaks:

$$\begin{aligned} \text{Loss of K.E. (of } A) &= \frac{1}{2} mu^2 - \frac{1}{2} mv^2 \\ &= \frac{1}{2} \times 2m \times 8.96 - 0 \end{aligned}$$

The motion must be considered in two parts, before and after the string breaks. The friction force acting on  $A$  is the same throughout the motion.

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# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 5

#### Question:

A car of mass 800 kg is travelling along a straight horizontal road. The resistance to motion from non-gravitational forces has a constant magnitude of 500 N. The engine of the car is working at a rate of 16 kW.

- a** Calculate the acceleration of the car when its speed is  $15 \text{ m s}^{-1}$ .

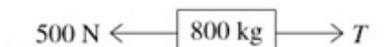
The car comes to a hill at the moment when it is travelling at  $15 \text{ m s}^{-1}$ . The road is still straight but is now inclined at  $5^\circ$  to the horizontal. The resistance to motion from non-gravitational forces is unchanged. The rate of working of the engine is increased to 24 kW.

- b** Calculate the new acceleration of the car.

#### Solution:

**a**

$$\longrightarrow 15 \text{ m s}^{-1} \quad \longrightarrow a \text{ m s}^{-2}$$



$$\text{Power} = F \times v$$

$$16\,000 = T \times 15$$

$$T = \frac{16\,000}{15}$$

$$F = ma$$

$$T - 500 = 800a$$

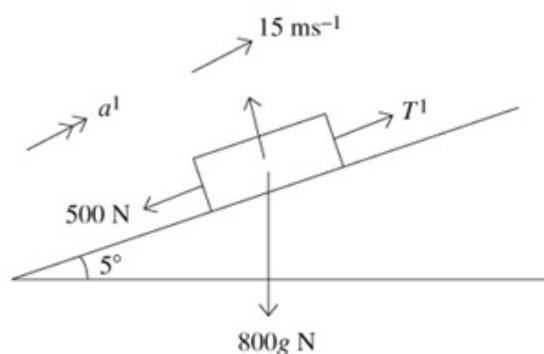
$$\frac{16\,000}{15} - 500 = 800a$$

$$a = \frac{\frac{16\,000}{15} - 500}{800}$$

$$a = 0.7083$$

The acceleration is  $0.708 \text{ m s}^{-2}$ .

Ensure units are consistent.

**b**

$$\text{Power} = F \times v$$

$$24\,000 = T' \times 15$$

$$T' = \frac{24\,000}{15}$$

$$R(\nearrow) F = ma$$

$$T' - 500 - 800g \sin 5^\circ = 800a$$

$$\frac{24\,000}{15} - 500 - 800 \times 9.8 \sin 5^\circ = 800a$$

$$800a = 416.698$$

$$a = 0.5208$$

The new acceleration is  $0.521 \text{ m s}^{-2}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 6

#### Question:

A car of mass 750 kg is moving at a constant speed of  $18 \text{ m s}^{-1}$  down a straight road inclined at an angle  $\theta$  to the horizontal, where  $\tan \theta = \frac{1}{20}$ . The resistance to motion from non-gravitational forces has a constant magnitude of 1000 N.

**a** Find, in kW, the rate of working of the car's engine.

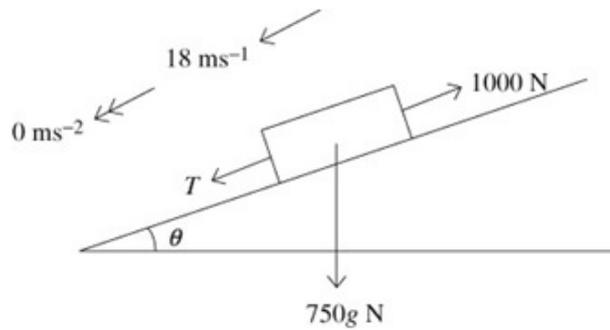
The engine of the car is now switched off and the car comes to rest  $T$  seconds later.

The resistance to motion from non-gravitational forces is unchanged.

**b** Find the value of  $T$ .

#### Solution:

a



$$\tan \theta = \frac{1}{20} \quad \theta = 2.8624^\circ$$

$$R(\nearrow) T + 750g \sin \theta = 1000$$

$$T = 1000 - 750 \times 9.8 \sin 2.8624^\circ$$

$$T = 632.95$$

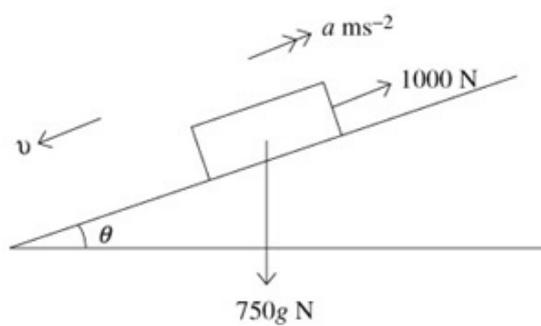
$$\text{Power} = F \times v$$

$$= 632.95 \times 18$$

$$= 11393 \text{ W}$$

The rate of working of the car's engine is 11.4 kW.

b



$$R(\nearrow) F = ma$$

$$1000 - 750 \times 9.8 \times \sin \theta = 750a$$

$$a = \frac{1000 - 750 \times 9.8 \sin 2.8624^\circ}{750}$$

$$a = 0.8439$$

$$u = 18 \text{ m s}^{-1}$$

$$v = 0 \text{ m s}^{-1}$$

$$a = -0.8439 \text{ m s}^{-2}$$

$$t = T$$

$$v = u + at$$

$$0 = 18 - 0.8439 \times T$$

$$T = \frac{18}{0.8439}$$

$$T = 21.32$$

The value of  $T$  is 21.3.

The tractive force is zero.

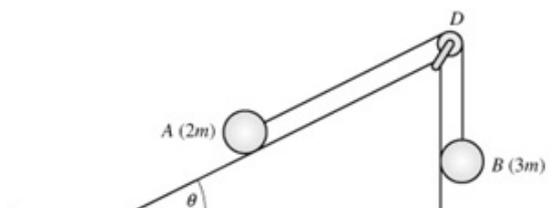
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## Solutionbank M2

### Edexcel AS and A Level Modular Mathematics

#### Exercise E, Question 7

#### Question:



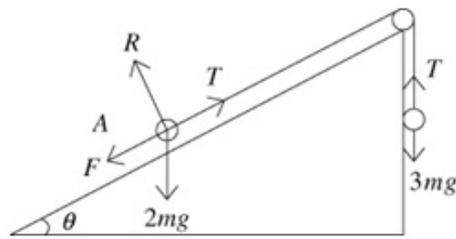
The diagram shows a particle  $A$  of mass  $2m$  which can move on the rough surface of a plane inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{3}{5}$ . A second particle  $B$  of mass  $3m$  hangs freely attached to a light inextensible string which passes over a smooth pulley fixed at  $D$ . The other end of the string is attached to  $A$ . The coefficient of friction between  $A$  and the plane is  $\frac{1}{4}$ . The system is released from rest with the string taut and  $A$  moves up a line of greatest slope of the plane. When each particle has moved a distance  $s$ ,  $A$  has not reached the pulley and  $B$  has not reached the ground.

- a Find an expression for the potential energy lost by the system when each particle has moved a distance  $s$ .

When each particle has moved a distance  $s$  they are moving with speed  $v$ .

- b Find an expression for  $v^2$ , in terms of  $s$ .

#### Solution:

**a**P.E. gained by  $A = mgh$ 

$$= 2mg \times (s \times \sin \theta)$$

$$= 2mg \times s \times \frac{3}{5}$$

$$= \frac{6mgs}{5}$$

P.E. lost by  $B = mgh$ 

$$= 3mgs$$

$$\therefore \text{P.E. lost by system} = 3mgs - \frac{6mgs}{5} = \frac{9mgs}{5}$$

**b** Consider  $A$ :

$$R(\sphericalangle) \quad R = 2mg \cos \theta$$

$$= 2mg \times \frac{4}{5}$$

$$= \frac{8mg}{5}$$

Friction is limiting:

$$F = \mu R$$

$$= \frac{1}{4} \times \frac{8mg}{5}$$

$$= \frac{2mg}{5}$$

Find the frictional force and use the work-energy principle.

Work done against friction =  $F \times s$ 

$$= \frac{2mgs}{5}$$

$$\text{K.E. gained by } A \text{ and } B = \frac{1}{2}(2m)v^2 + \frac{1}{2}(3m)v^2$$

$$= \frac{5mv^2}{2}$$

Work-energy principle:

K.E. gained + work done against friction = P.E. lost

$$\frac{5mv^2}{2} + \frac{2mgs}{5} = \frac{9mgs}{5}$$

$$\frac{5mv^2}{2} = \frac{7mgs}{5}$$

$$v^2 = \frac{2 \times 7mgs}{5 \times 5m}$$

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# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 8

#### Question:

A parcel of mass 5 kg is resting on a platform inclined at  $25^\circ$  to the horizontal.

The coefficient of friction between the parcel and the platform is 0.3. The parcel is released from rest and slides down a line of greatest slope of the platform. Calculate

- the speed of the parcel after it has been moving for 2 s,
- the potential energy lost by the parcel during this time.

#### Solution:



$$\begin{aligned} \text{Work done} &= 18 \cos 25^\circ \times 14 \\ &= 228.3 \end{aligned}$$

← Work done = *horizontal* component of tension  $\times$  *horizontal* distance moved

The work done is 228 J.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

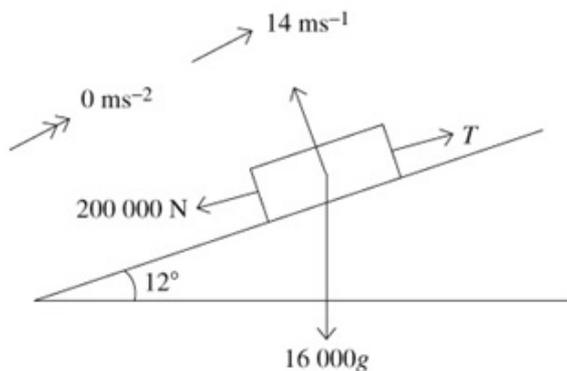
### Exercise E, Question 9

#### Question:

A lorry of mass  $16\,000\text{ kg}$  is travelling up a straight road inclined at  $12^\circ$  to the horizontal.

The lorry is travelling at a constant speed of  $14\text{ m s}^{-1}$  and the resistance to motion from non-gravitational forces has a constant magnitude of  $200\text{ kN}$ . Find the work done in  $10\text{ s}$  by the engine of the lorry.

#### Solution:



$$R(\nearrow) T = 200\,000 + 16\,000g \sin 12^\circ$$

$$T = 232\,600$$

Work done in  $10\text{ s} = \text{force} \times \text{distance moved}$

$$= 232\,600 \times (14 \times 10)$$

$$= 32\,564\,000$$

The work done in  $10\text{ s}$  is  $32\,600\,000\text{ J}$  (or  $32\,600\text{ kJ}$ ).

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 10

#### Question:

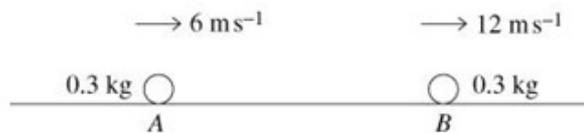
A particle  $P$  of mass  $0.3 \text{ kg}$  is moving in a straight line on a smooth horizontal surface under the action of a constant horizontal force. The particle passes point  $A$  with speed  $6 \text{ m s}^{-1}$  and point  $B$  with speed  $12 \text{ m s}^{-1}$ .

- Find the kinetic energy gained by  $P$  while moving from  $A$  to  $B$ .
- Write down the work done by the constant force.

The distance from  $A$  to  $B$  is  $4 \text{ m}$ .

- Calculate the magnitude of the force.

#### Solution:



**a**

$$\begin{aligned} \text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 0.3 \times 12^2 - \frac{1}{2} \times 0.3 \times 6^2 \\ &= 16.2 \end{aligned}$$

The K.E. gained is  $16.2 \text{ J}$ .

**b**

The work done by the force is  $16.2 \text{ J}$ .

**c**

$$\text{Work done} = F \times s$$

$$16.2 = F \times 4$$

$$F = \frac{16.2}{4}$$

$$F = 4.05$$

The force has magnitude  $4.05 \text{ N}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

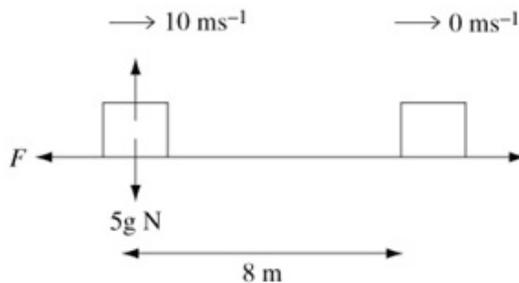
### Exercise E, Question 11

#### Question:

A box of mass 5 kg slides in a straight line across a rough horizontal floor. The initial speed of the box is  $10 \text{ m s}^{-1}$ . The only resistance to the motion is the frictional force between the box and the floor. The box comes to rest after moving 8 m. Calculate

- the kinetic energy lost by the box in coming to rest,
- the coefficient of friction between the box and the floor.

#### Solution:



**a**

$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 5 \times 10^2 - 0 \\ &= 250 \end{aligned}$$

The K.E. lost is 250 J.

**b** Work done against friction = 250 J

$$\text{Work done} = F \times s$$

$$250 = F \times 8$$

$$F = \frac{250}{8}$$

$$R(\uparrow) R = 5g$$

$$\text{Friction is limiting } F = \mu R$$

$$\frac{250}{8} = \mu \times 5g$$

$$\mu = \frac{250}{8 \times 5 \times 9.8} = 0.6377$$

The coefficient of friction is 0.638.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 12

**Question:**

A car of mass 900 kg is moving along a straight horizontal road. The resistance to motion has a constant magnitude. The engine of the car is working at a rate of 15 kW. When the car is moving with speed  $20 \text{ m s}^{-1}$ , the acceleration of the car is  $0.3 \text{ m s}^{-2}$ .

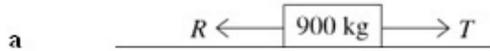
**a** Find the magnitude of the resistance.

The car now moves downhill on a straight road inclined at  $4^\circ$  to the horizontal. The engine of the car is now working at a rate of 8 kW. The resistance to motion from non-gravitational forces remains unchanged.

**b** Calculate the speed of the car when its acceleration is  $0.5 \text{ m s}^{-2}$ .

**Solution:**

$$\longrightarrow 20 \text{ m s}^{-1} \quad \longrightarrow\!\!\! \longrightarrow 0.3 \text{ m s}^{-2}$$



$$\text{Power} = F \times v$$

$$T = \frac{15000}{20} = 750$$

$$F = ma$$

$$T - R = 900 \times 0.3$$

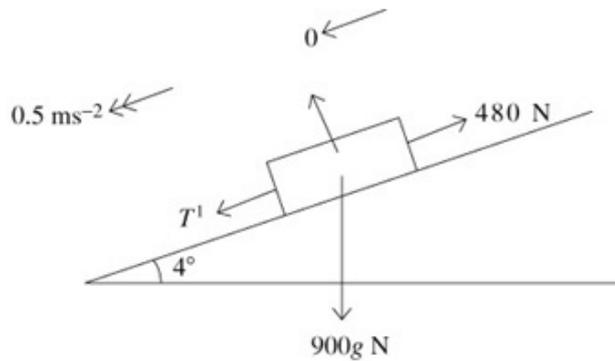
$$750 - R = 270$$

$$R = 750 - 270$$

$$R = 480$$

The magnitude of the resistance is 480 N.

**b**



$$F = ma$$

$$T' + 900g \sin 4^\circ - 480 = 900 \times 0.5$$

$$T' = 450 + 480 - 900g \sin 4^\circ$$

$$\text{Power} = F \times v$$

$$8000 = (450 + 480 - 900g \sin 4^\circ)v$$

$$v = \frac{8000}{(450 + 480 - 900g \sin 4^\circ)}$$

$$v = 25.41$$

The speed of the car is 25.4 m s<sup>-1</sup>.

## Solutionbank M2

### Edexcel AS and A Level Modular Mathematics

#### Exercise E, Question 13

#### Question:

A block of wood of mass 4 kg is pulled across a rough horizontal floor by a rope inclined at  $15^\circ$  to the horizontal. The tension in the rope is constant and has magnitude 75 N.

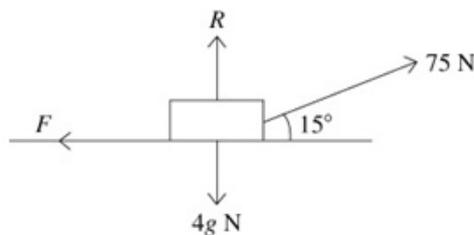
The coefficient of friction between the block and the floor is  $\frac{3}{8}$ .

- Find the magnitude of the frictional force opposing the motion.
- Find the work done by the tension when the block moves 6 m.

The block is initially at rest.

- Find the speed of the block when it has moved 6 m.

#### Solution:



**a**

$$R(\uparrow) + 75 \sin 15^\circ = 4g$$

$$R = 4g - 75 \sin 15^\circ$$

Friction is limiting:  $F = \mu R$

$$F = \frac{3}{8} \times (4 \times 9.8 - 75 \sin 15^\circ)$$

$$F = 7.420$$

The magnitude of the frictional force is 7.42 N.

**b**

$$\text{Work done} = F \times s$$

$$= 75 \cos 15^\circ \times 6$$

$$= 434.66$$

The work done is 435 J.

**c**

K.E. gained = work done by tension - work done against friction

$$\frac{1}{2} \times 4v^2 = 434.66 - 7.42 \times 6$$

$$v^2 = \frac{1}{2}(434.66 - 7.42 \times 6)$$

$$v = 13.96$$

The block is moving at  $14.0 \text{ m s}^{-1}$ .

Use the work-energy principle.

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 14

#### Question:

The engine of a lorry works at a constant rate of 20 kW. The lorry has a mass of 1800 kg.

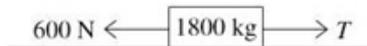
When moving along a straight horizontal road there is a constant resistance to motion of magnitude 600 N. Calculate

- the maximum speed of the lorry,
- the acceleration of the lorry, in  $\text{m s}^{-2}$ , when its speed is  $20 \text{ m s}^{-1}$ .

#### Solution:

a

$$\longrightarrow v \text{ m s}^{-1} \quad \longrightarrow\!\!\!\!> 0 \text{ m s}^{-2}$$



$$R(\rightarrow) T = 600$$

Ensure units are consistent.

$$\text{Power} = F \times v$$

$$20\,000 = 600v$$

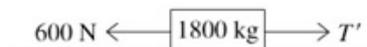
$$v = \frac{20\,000}{600}$$

$$v = 33.33$$

The lorry's maximum speed is  $33.3 \text{ m s}^{-1}$ .

b

$$\longrightarrow 20 \text{ m s}^{-1} \quad \longrightarrow\!\!\!\!> a \text{ m s}^{-2}$$



$$\text{Power} = F \times v$$

$$20\,000 = T' \times 20$$

$$T' = 1000$$

$$F = ma$$

$$T' - 600 = 1800a$$

$$1000 - 600 = 1800a$$

$$a = \frac{400}{1800}$$

$$a = 0.2222$$

The acceleration is  $0.222 \text{ m s}^{-2}$ .

# Solutionbank M2

## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 15

#### Question:

A car of mass 1200 kg is travelling at a constant speed of  $20 \text{ m s}^{-1}$  along a straight horizontal road. The constant resistance to motion has magnitude 600 N.

- a** Calculate the power, in kW, developed by the engine of the car.

The rate of working of the engine of the car is suddenly increased and the initial acceleration of the car is  $0.5 \text{ m s}^{-2}$ . The resistance to motion is unchanged.

- b** Find the new rate of working of the engine of the car.

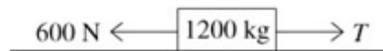
The car now comes to a hill. The road is still straight but is now inclined at  $20^\circ$  to the horizontal. The rate of working of the engine of the car is increased further to 50 kW.

The resistance to motion from non-gravitational forces still has magnitude 600 N. The car climbs the hill at a constant speed  $V \text{ m s}^{-1}$ .

- c** Find the value of  $V$ .

#### Solution:

$$\longrightarrow 20 \text{ m s}^{-1} \quad \longrightarrow\!\!\! \gg 0 \text{ m s}^{-2}$$



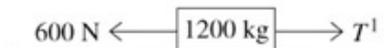
**a**  $R(\rightarrow)T = 600$

$$\begin{aligned} \text{Power} &= F \times v \\ &= 600 \times 20 \\ &= 12\,000 \text{ W} \\ &= 12 \text{ kW} \end{aligned}$$

The power is 12 kW.

**b**

$$\longrightarrow 20 \text{ m s}^{-1} \quad \longrightarrow\!\!\! \gg 0.5 \text{ m s}^{-2}$$



$$F = ma$$

$$T' - 600 = 1200 \times 0.5$$

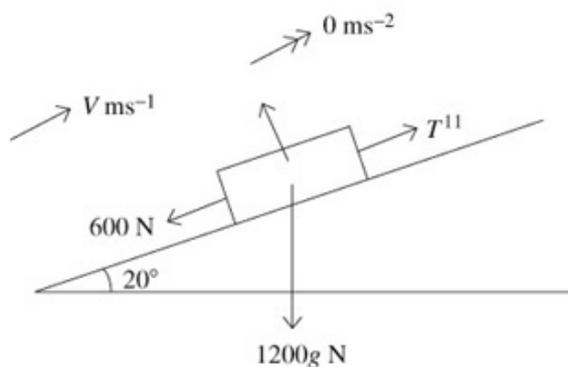
$$T' = 600 + 600$$

$$T' = 1200$$

$$\begin{aligned} \text{Power} &= F \times v \\ &= 1200 \times 20 \\ &= 24\,000 \end{aligned}$$

The new rate of working is 24 kW.

**c**



$$R(\nearrow)T'' = 600 + 1200g \sin 20^\circ$$

$$\text{Power} = F \times v$$

$$50\,000 = (600 + 1200g \sin 20^\circ)V$$

$$V = \frac{50\,000}{(600 + 1200g \sin 20^\circ)}$$

$$V = 10.82$$

$$= 10.8$$

